S-parameter Analysis Examples

Mason's Non-Touching Loop Rule

First identify the input and output nodes you are interested in. For example, to calculate S_{11} of a network you need to concentrate on the nodes a1 (input) and b1 (output)

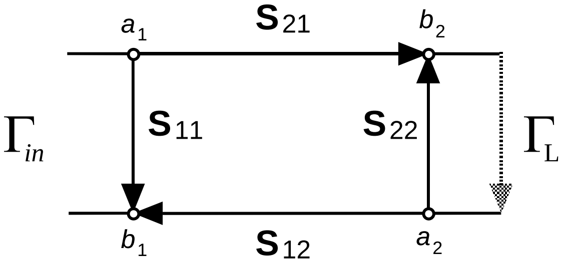
Second, identify all the PATHS going from the input to output nodes.

Third, identify any loops in the network, and note whether they touch any of your paths.

Fourth, apply the following equation:

$$Transfer\ \ Function = \frac{Output\ Variable}{Input\ Variable} = \frac{\sum_{n=1,2,3}^{\infty} P_n \cdot \begin{pmatrix} 1-\Sigma 1\text{st order loops not touching } P_n \\ +\Sigma 2nd \text{ order loops not touching } P_n \\ -\Sigma 3rd \text{ order loops not touching } P_n \\ +\ldots \end{pmatrix}}{\begin{matrix} 1-\Sigma 1\text{st order loops} \\ +\Sigma 2nd \text{ order loops} \\ -\Sigma 3rd \text{ order loops} \\ +\ldots \end{matrix}}$$

1. Terminated 2-Port Network



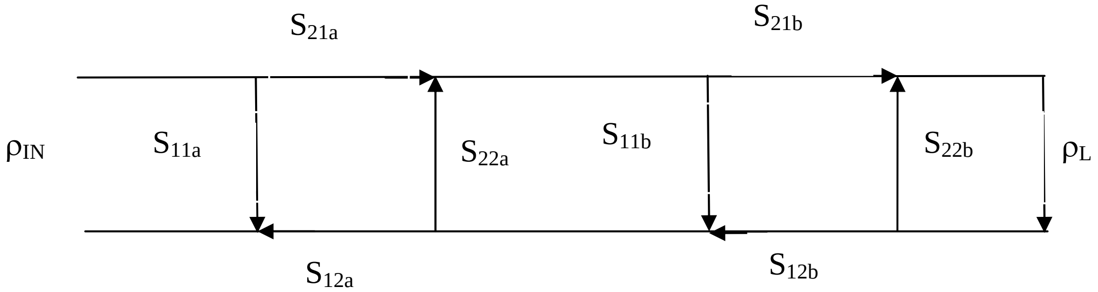
$$b_1 = S_{11}a_1 + S_{12}\left(\Gamma_L b_2\right)$$
$$b_2 = S_{21}a_1 + S_{22}\left(\Gamma_L b_2\right)$$
$$\therefore b_2 = \left(1-\Gamma_L S_{22}\right) = S_{21}a_1$$
$$\therefore \Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}$$

If the 2-port network is terminated with a load impedance having a voltage-wave reflection coefficient $\Gamma_L = 0.5$, determine if the overall one-port network is stable.

If $\left|\Gamma_{in}\right| \le 1$, the circuit will be unconditionally stable. Since $S_{11} = S_{22} = 0$, $\left|\Gamma_{in}\right| \le S_{21}S_{12} = 0.07e^{-j\pi/2}$ and $\left|\Gamma_{in}\right| = 0.07/2 << 1$, i.e. unconditionally stable!

2. Cascaded Topologies

Two amplifiers are connected in cascade, with the final output stage terminated with a load impedance having an arbitrary value. The amplifiers can be defined by the S-parameter matrices [S_a] and [S_b], and the load impedance can be expressed by its voltage-wave reflection coefficient ρ_L. Draw the corresponding signal flow graph and determine the exact expression for the overall input voltage-wave reflection coefficient ρ_{IN}.



Paths (P) from nodes a1 to b1

P1 = S11a

P2 = S21a S21b ρ_L S12b S12a

P3 = S21a S11b S12a

First Order Loops (L)

FL1 = S22b ρ_L not touching P1 or P3

FL2 = S21b ρ_L S12b S22a not touching P1

FL3 = S22a S11b not touching P1

Second Order loop

SL = FL1 FL3

$$\rho_{IN} = b1/a1 = \frac{P1[1 - (FL1 + FL2 + FL3) + SL] + P2\,[1-0] + P3[1 - FL1]}{1 - (FL1 + FL2 + FL3) + SL}$$

For simplicity, phase angle is ignored and the amplifiers are assumed to be identical, with the following 2-port S-Parameters:

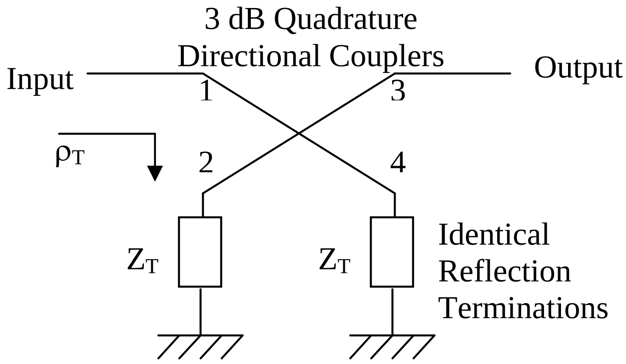
$$[S_a] = [S_b] = \begin{bmatrix} 0.1 & 0.1 \\ 10 & 0.1 \end{bmatrix} \quad \textit{and} \quad \rho_L = 0.5$$

Calculate the overall input return loss for the circuit in.

ρ_{IN} = 0.68405/0.8905 = 0.76816 and overall Input Return Loss is -2.291 dB.

The introduction of the amplifiers has degraded the overall Input Return Loss from the value of Load Return Loss = -6 dB.

3. Lossless Single-stage Reflection Topology



$$S_{21}\,|_{OVERALL} = S_{21}\rho_T S_{41} + S_{41}\rho_T S_{21} = 2S_{21}\rho_T S_{41}$$

For a 3 dB quadrature coupler:

$$S_{41} = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{2}} \quad and \quad S_{21} = \frac{1}{\sqrt{2}}e^{-j0}$$

$$\therefore S_{21}\,|_{OVERALL} = 2\frac{1}{\sqrt{2}}e^{-j\frac{\pi}{2}}\rho_T\frac{1}{\sqrt{2}} = \rho_T e^{-j\frac{\pi}{2}}$$

The reflection topology transforms a reflection coefficient into a transmission coefficient.

<u>Reflection-type Attenuator</u>
Here the reflection coefficient is implemented with a PIN diode or cold-FET, to realise a variable resistance, R_T.

$$\rho_T(V) = \frac{R_T(V) - Zo}{R_T(V) + Zo} \equiv |\rho_T(V)|$$

$$\therefore S_{21}\,|_{OVERALL} = |\rho_T(V)|e^{-j\frac{\pi}{2}}$$

Therefore, the relative attenuation, $\Delta\,|\,S_{21}\,|_{OVERALL} = \Delta|\rho_T(V)|$

<u>Reflection-type Phase Shifter</u>
Here the reflection coefficient is implemented with a varactor diode, to realise a variable capatitance, $C_T = \frac{-1}{\omega X_T}$.

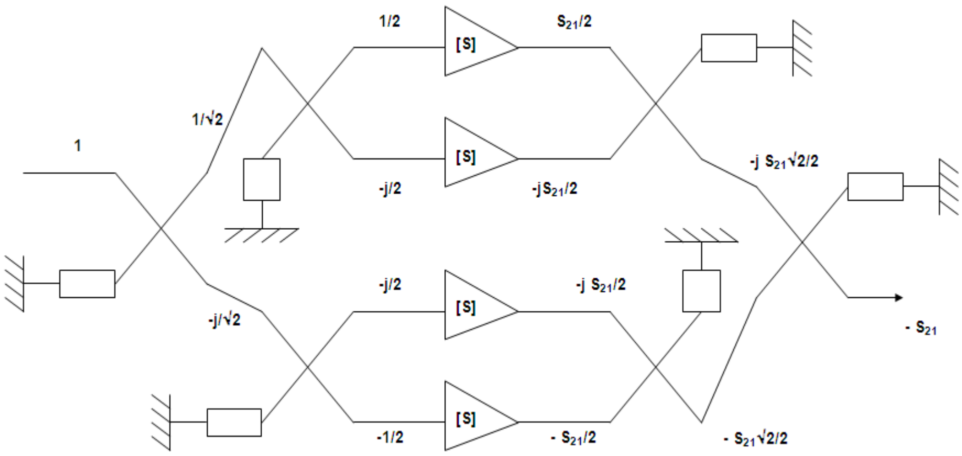
$$\rho_T(V) = \frac{X_T(V) - Zo}{X_T(V) + Zo} \equiv 1.e^{j\angle\rho_T(V)}$$

$$\therefore S_{21}\,|_{OVERALL} = 1.e^{j\angle\rho_T(V) - j\frac{\pi}{2}}$$

Therefore, the relative phase shift, $\Delta\angle S_{21}\,|_{OVERALL} = \Delta\angle\rho_T(V)$

4. Tandem Topologies

Consider the double-balanced amplifier. If 3 dB quadrature couplers are used in conjunction with identical non-ideal single-ended amplifiers, use S-parameter analysis to determine expressions for the overall insertion gain and input return loss. Assume the couplers are perfectly matched to the reference impedance, Zo, and the interconnections between the main components are ideal.



$$S_{21}\,|_{overall} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{21}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{21}\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) +$$

$$\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{21}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{21}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = -S_{21}$$

$$Insertion\ Loss = 10\log\left\{|\,S_{21}\,|^2\right\}$$

$$S_{11}\,|_{overall} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{11}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{11}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) +$$

$$\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)S_{11}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) + \left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right)S_{11}\left(\frac{-j}{\sqrt{2}}\right)\left(\frac{-j}{\sqrt{2}}\right) = 0$$

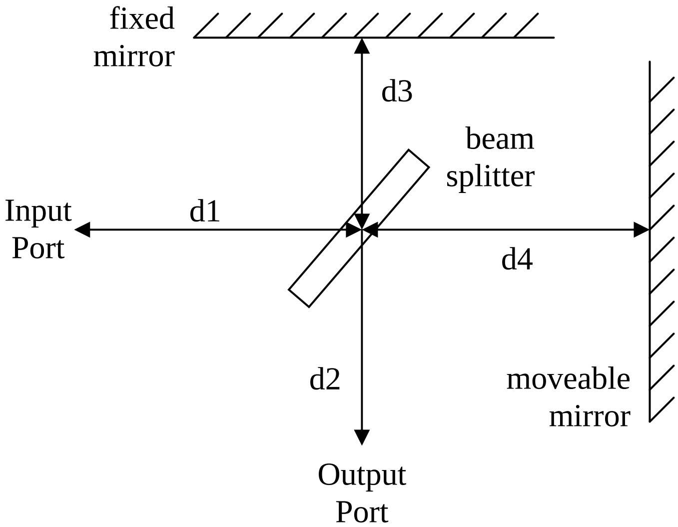
$$\mathrm{Re}\,turn\ Loss = 10\log\left\{|\,0\,|^2\right\} \rightarrow -\infty$$

The main application of this amplifier is power combining, since the output power is ideally a factor of 4 greater than that of the single-ended amplifier.

This topology is also robust to single failure. For example, if the working single-ended amplifiers have a forward voltage wave transmission coefficient of $S_{21} = |10|\angle 35°$, determine the overall insertion gain and input return loss if one of the amplifiers fails, such that $S_{21} = 0$. Assume that there is no change in the input or output impedances of the failed transistor. If one of the single-ended amplifiers fails then $S_{21}|_{overall} = 3S_{21}/4$ and Insertion gain is $10\log\{(3|S_{21}|/4)^2\} = 17.5$ dB, i.e. a drop of 2.5 dB from a fully working amplifier. The input return loss should not change if the impedance of the failed amplifier does not change and is, therefore, still minus infinity.

Therefore, this type of power combining amplifier is useful because it provides redundancy in the case of failure and also ideal port impedance matching. The disadvantages of this topology is that it requires 4 identical single-ended amplifiers. Also, practical losses in the couplers result in a direct loss in power gain and output efficiency will be significantly reduced.

5. Michelson interferometer



The Michelson interferometer can be analysed as a general two-port network. By inspection, write down equations for the effective forward voltage wave transmission coefficient (S_{21}) and input voltage wave reflection coefficient (S_{11}) for this passive and reciprocal network. Clearly define all variables used. Hints, the electrical path lengths can be represented by $(k_o dx)$, where $k_o = 2\pi/\lambda$, λ is the wavelength for a monochromatic input source, x is the designation of a particular path and the beam splitter is both symmetrical and reciprocal.

The effective input voltage wave transmission coefficient is given by:

$$S_{21} = \exp(-jk_o d1)\rho_s \exp(-jk_o d3)\rho_m \exp(-jk_o d3)\tau_s \exp(-jk_o d2)$$
$$+\exp(-jk_o d1)\tau_s \exp(-jk_o d4)\rho_m \exp(-jk_o d4)\rho_s \exp(-jk_o d2)$$

$\rho_s =$ voltage wave reflection coefficient of the beam splitter
$\tau_s =$ voltage wave transmission coefficient of the beam splitter
$\rho_m =$ voltage wave reflection coefficient of a mirror

$$S_{11} = \exp(-jk_o d1)\rho_s \exp(-jk_o d3)\rho_m \exp(-jk_o d3)\rho_s$$
$$+\exp(-jk_o d1)\tau_s \exp(-jk_o d4)\rho_m \exp(-jk_o d4)\tau_s$$

Given that the optical path difference is given by $\delta = 2(d3 - d4)$, if $d1 = d2 = \lambda$ and both mirrors are made from perfectly conducting metals, simplify the equations obtained:

Given:
$d1 = d2 = \lambda \quad \therefore \exp(-jk_o d1) = \exp(-jk_o d2) = \exp(-j2\pi) = 1$
And the mirrors are made from perfectly conducting metal, $\therefore \rho_m = -1$

$$S_{21} = -\rho_s \tau_s [\exp(-j2k_o d3) + \exp(-j2k_o d4)]$$

$$S_{11} = -\rho_s{}^2 \exp(-j2k_o d3) - \tau_s{}^2 \exp(-j2k_o d4)$$

For this interferometer to function properly, an ideal beam splitter must reflect 50% of any incident power and allow the rest of the power to be transmitted through without attenuation.

Write down the effective forward voltage wave transmission and forward voltage wave reflection coefficients for an ideal beam splitter, given that they must be in phase quadrature with one another. Hint, there are a number of possible solutions, so only choose one.

Since 50% of the power is reflected and 50% is transmitted through the ideal beam splitter then it must have the following solutions for its forward voltage wave transmission coefficient and input voltage wave reflection coefficient:

$$\tau_s = \rho_s \exp(\pm j\pi/2) = \frac{\exp[j(\angle\rho_s \pm \pi/2)]}{\sqrt{2}} \quad e.g.\ \tau_s = \pm\frac{j}{\sqrt{2}}$$

$$\rho_s = \tau_s \exp(\pm j\pi/2) = \frac{\exp[j(\angle\tau_s \pm \pi/2)]}{\sqrt{2}} \quad e.g.\ \rho_s = \pm\frac{1}{\sqrt{2}}$$

From this solution, show that the beam splitter obeys the conservation of energy principle.

For a lossless 2-port network, the beam splitter must satisfy the following in order to obey the conservation of energy principle:

$|\tau_s|^2 + |\rho_s|^2 = 1$ and this is confirmed!

Also, simplify the previous equations obtained for the 2-port network:

$$S_{21} = \pm\frac{j}{2}\exp(-j2k_o d3)[1+\exp(+jk_o\delta)]$$

$$S_{11} = \pm\frac{1}{2}\exp(-j2k_o d3)[1-\exp(+jk_o\delta)]$$

From this solution, show that this Michelson interferometer obeys the conservation of energy principle for $\delta = n\lambda$ and $(n+1/2)\lambda$, where n is any positive integer.

For a lossless 2-port network, the interferometer must satisfy the following in order to obey the conservation of energy principle:

$|S_{21}|^2 + |S_{11}|^2 = 1$ and this is confirmed, since:

$|S_{21}| = 1$ and $|S_{11}| = 0$ when $\delta = n\lambda$

$|S_{21}| = 0$ and $|S_{11}| = 1$ when $\delta = (n+1/2)\lambda$