Lecture #6

Engineering Approach for Analytical Electromagnetic Modelling of THz Metal Structures

The main objective of this lecture is to teach a useful tool for enabling a student to derive analytical expressions for a number of electromagnetic problems relating to THz metal structures. The traditional approach when using the classical relaxation-effect model can be mathematically cumbersome and not insightful. This lecture briefly introduces various interrelated electrical engineering concepts as tools for characterizing the intrinsic frequency dispersive nature of normal metals at room temperature. This Engineering Approach dramatically simplifies the otherwise complex analysis and allows for a much deeper insight to be gained into the classical relaxation-effect model. Problems with worked-though solutions will be given in class.

Drude's Model

Classical relaxation-effect frequency dispersion model:

$$\sigma_R \equiv \sigma_R' - j\sigma_R'' = \frac{\sigma_o}{(1 + j\omega\tau)} \Rightarrow \begin{cases} \sigma_R' = \dfrac{\sigma_o}{1+(\omega\tau)^2} & Simple\ \ Relaxation\ -Effect\ \ Model \\ \sigma_o & Classical\ \ Skin-Effect\ \ Model \end{cases}$$

OVERVIEW

- Drude's frequency dispersion model
- The Engineering Approach
  - Equivalent transmission line model
  - Q-factor for metals
  - Kinetic inductance
  - Complex skin depth
  - Boundary resistance coefficient
- Applications
  - Metal-pipe rectangular waveguides
  - Cavity resonators
  - Single metal planar shield
- Conclusions

#1 Line Modelling

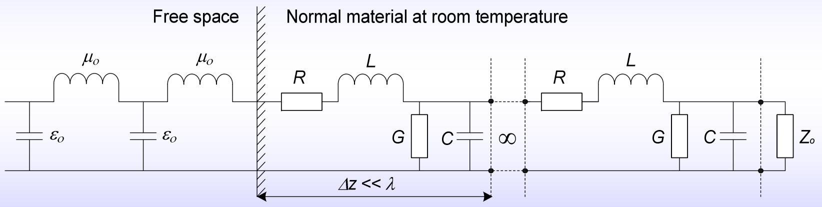
Electromagnetic Characterisation of Homogenous Materials

$$\eta_I = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{(\omega\mu_o\mu_r'') + j\omega(\mu_o\mu_r')}{(\sigma' + \omega\varepsilon_o\varepsilon_r'') + j\omega\left(\dfrac{-\sigma''}{\omega} + \varepsilon_o\varepsilon_r'\right)}} \quad [\Omega]$$

$$\gamma \equiv \frac{j\omega\mu}{\eta_I} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{[(\omega\mu_o\mu_r'') + j\omega(\mu_o\mu_r')]\left[(\sigma' + \omega\varepsilon_o\varepsilon_r'') + j\omega\left(\frac{-\sigma''}{\omega} + \varepsilon_o\varepsilon_r'\right)\right]} \quad [m^{-1}]$$

Compare vector Helmholtz equations (for an unbounded plane wave in one-dimensional space) with those of the telegrapher's equations.

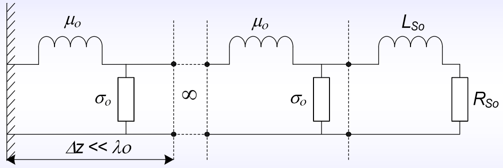
Generic Equivalent Transmission Line Model



Classical skin-effect model:

$$\eta_{Io} = \sqrt{\frac{j\omega\mu_o}{\sigma_o}} = \sqrt{\frac{\omega\mu_o}{2\sigma_o}}(1+j)[\Omega]$$

$$Z_{So} \equiv (R_{So} + jX_{So})[\Omega / square]$$



Intrinsic Modelling for Normal Metals at Room Temperature

Generic equations:

$$Z_S \equiv R_S + jX_S = \sqrt{\frac{j\omega\mu_0\mu_r}{\sigma + j\omega\varepsilon_o}} \cong \sqrt{\frac{j\omega\mu_0\mu_r}{\sigma}} \quad \textbf{with} \quad \omega < 10^{15}\ rad/s$$

$$\gamma_S \equiv \alpha_S + j\beta_S = \frac{j\omega\mu_0\mu_r}{Z_S} = \sqrt{j\omega\mu_o\mu_r\sigma} \quad and \quad \delta_S = \frac{1}{\Re\{\gamma_S\}} = \frac{1}{\alpha_S} \quad and \quad \delta_c \equiv \delta_c' - j\delta_c'' = \frac{1}{\gamma_S}$$

Classical relaxation-effect model:

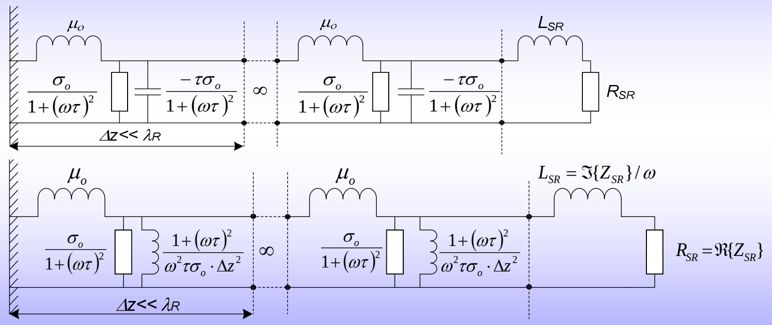
$$Zo_R = \sqrt{\frac{R_R + j\omega L_R}{G_R + j\omega C_R}} \qquad \gamma_R \equiv \frac{R_R + j\omega L_R}{Zo_R} = \sqrt{(R_R + j\omega L_R)(G_R + j\omega C_R)}$$

$$Zo_R \Rightarrow \eta_{IR} \cong \sqrt{\frac{j\omega\mu_o}{\left[\frac{\sigma_o}{1+(\omega\tau)^2}\right] + j\omega\left[\frac{-\tau\sigma_o}{1+(\omega\tau)^2}\right]}} \Rightarrow Z_{SR} \equiv (R_{SR} + jX_{SR})$$

$$R_{SR} \equiv \Re\{Z_{SR}\} = \frac{R_{So}}{(1+\xi\omega\tau)} \qquad L_{SR} \equiv \frac{\Im\{Z_{SR}\}}{\omega} = \frac{R_{So}}{\omega}(1+\xi\omega\tau)$$

$$\xi = \sqrt{\sqrt{u^{-4} + u^{-2}} + u^{-1} - u^{-1}} \qquad u = (\omega\tau)$$

Classical relaxation-effect models:



Radio Frequency Engineering
Lecture #6 Engineering Approach

Propagation Delay Per Unit Wavelength

$$\tau_{pR} = \frac{l}{v_{pR}} \to \frac{\lambda_R}{v_{pR}} = \frac{1}{f} \quad [s/\lambda_R] \quad where \quad \lambda_R = \frac{2\pi}{\Im\{\gamma_R\}} \; ; \; v_{pR} = \frac{\omega}{\Im\{\gamma_R\}}$$

$$\gamma_R = \sqrt{j\omega L_R(G_R + j\omega C_R)} = \sqrt{j\omega L_R\left(G_R - j\frac{1}{\omega L_{SHUNT\_R}\,\Delta z^2}\right)} \equiv \sqrt{j\omega\mu\sigma_R}$$

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Radio Frequency Engineering
Lecture #6 Engineering Approach

Distributed-element parameters for the classical relaxation-effect model

$$R_R = 0 \quad ; \quad L_R = \mu_o \quad ; \quad G_R = \frac{\sigma_o}{1+(\omega\tau)^2} \quad ; \quad C_R = \frac{-\tau\sigma_o}{1+(\omega\tau)^2} + \varepsilon_o \cong \frac{-\tau\sigma_o}{1+(\omega\tau)^2}$$

Distributive shunt inductance

$$L_{SHUNT\_R} \equiv \frac{-1}{\omega^2 C_R \Delta z^2} = \frac{1+(\omega\tau)^2}{\omega^2\tau\sigma_o\Delta z^2} \quad [H/m]$$

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Radio Frequency Engineering
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Circuit-based Simulation Results for Gold at Room Temperature

with $\omega\tau = 1$, depth $l = \lambda_R$, $\Delta z = \lambda_R / 400 = 1.067$ [nm]

Equivalent Transmission Line model	Elementary Lumped-element Circuit Values					Z_{IN} [Ω] (Theory: 0.4608+j1.1124)	τ_{pR} [fs/λ_R] (Theory: 170:494)
	$R_R\cdot\Delta z$ [Ω]	$L_R\cdot\Delta z$ [fH]	$G_R\cdot\Delta z$ [mS]	$C_R\cdot\Delta z$ [fF]	$L_{SHUNT\_R}\cdot\Delta z$ [pH]	$Z_T = Z_{SR}$	
Extracted model	—	1.341	24.1	-0.654	—	0.4607 + j1.137	170.491
Alternative Extracted model	—	1.341	24.1	—	1.126	0.4607 + j1.137	170.491

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#2 Kinetic Inductance

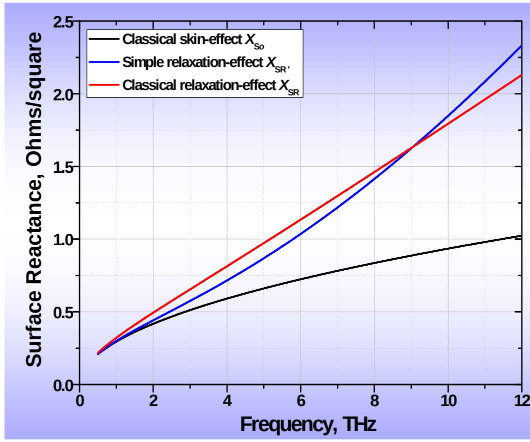
$$L_{SR} = L_{So} + \xi\, L_k = L_{So}(1 + \xi\,\omega\tau) \quad [H/square]$$

Kinetic inductance is created from the inertial mass of a mobile charge carrier distribution within an alternating electric field (i.e. classical electrodynamics).

$$L_{So} = \mu\left(\frac{\delta_{So}}{2}\right) \propto (\omega\tau)^{-1/2} \quad \text{and} \quad L_k = \frac{\tau}{\sigma_o \delta_{So}} = \propto (\omega\tau)^{+1/2}$$

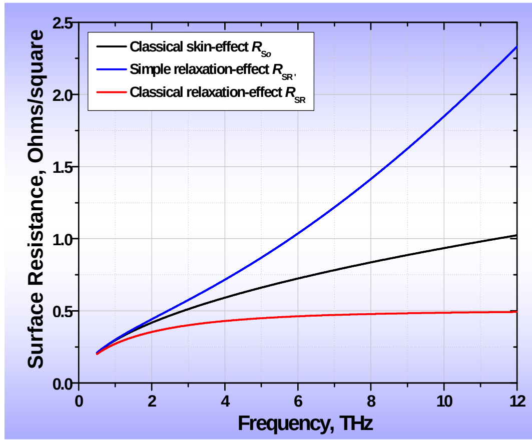
$$R_{SR} = \frac{L_{So}}{L_{SR}}\left(\frac{L_k}{\tau}\right) \quad \text{and} \quad X_{SR} = \frac{L_{SR}}{L_{So}}\left(\frac{L_k}{\tau}\right)$$

$$\xi \approx a = 0.539 \quad for\ 0 \le \omega\tau \le 2$$



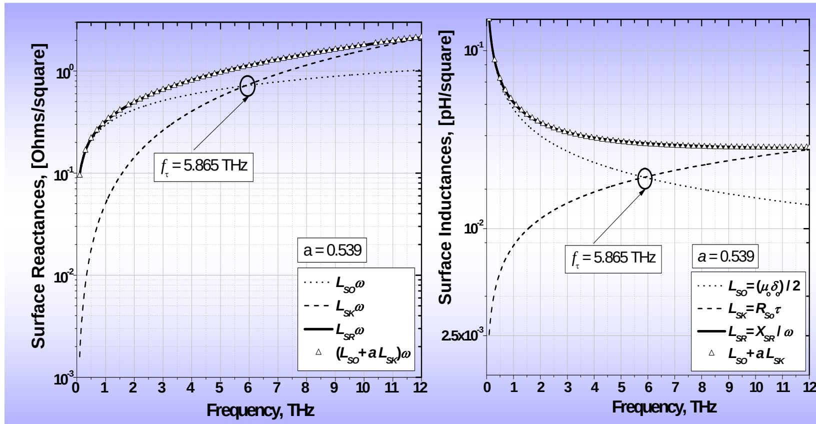
Gold at room temperature

Intrinsic classical skin-effect model underestimates perturbation detuning



Gold at room temperature

Intrinsic classical skin-effect model overestimates ohmic losses and Detuning. Also, underestimates extrinsic losses.



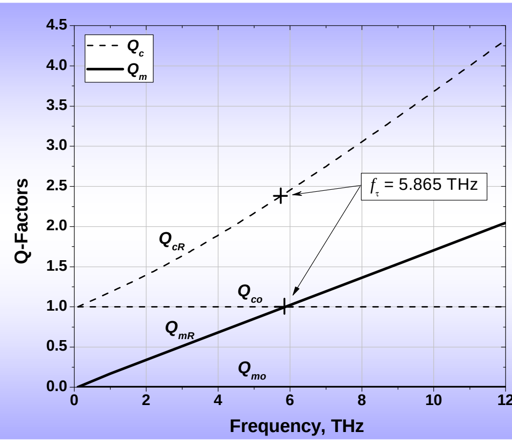
#3 Q-factor

Material Q-Factor

$$Q_m = \frac{1}{\tan\delta} = \left|\frac{\Re\{\varepsilon_{r\ effective}\}}{\Im\{\varepsilon_{r\ effective}\}}\right|$$

$$Q_m = \frac{\Re\{n^2\}}{\Im\{n^2\}} = \frac{\Re\{\gamma^2\}}{\Im\{\gamma^2\}} = \frac{\Im\{\sigma_{equivalent}\}}{\Re\{\sigma_{equivalent}\}} = \frac{-\Re\{Z_S{}^2\}}{\Im\{Z_S{}^2\}} \Rightarrow \begin{cases} \equiv 0 & for\ Q_{mo} \\ > 0 & for\ Q_{mR} \end{cases}$$

Gold at room temperature



Component Q-Factor

$$Q_c = \frac{|X_c|}{R_c}$$

$$Q_c = \frac{\Im\{n\}}{\Re\{n\}} = \frac{\Re\{\gamma\}}{\Im\{\gamma\}} = \frac{\Im\{Z_S\}}{\Re\{Z_S\}} \Rightarrow \begin{cases} \equiv 1 & for\ Q_{co} \\ > 1 & for\ Q_{cR} \end{cases}$$

$$Q_m = \left|\frac{1 - Q_c{}^2}{2Q_c}\right| \quad \text{and} \quad Q_c = Q_m + \sqrt{1 + Q_m{}^2}$$

Classical relaxation-effect model:

$$Q_{mR} \equiv (\omega\tau) \qquad Q_{cR} \equiv \left(1 + \xi\, Q_{mR}\right)^2$$

#4 Complex Skin Depth

Complex Skin Depth: $\delta_c = \frac{1}{\gamma} \equiv \delta_c' - j\delta_c''[m]$ $\qquad Q_c = \frac{\Re\{\delta_c\}}{\Im\{\delta_c\}}$

$$\hat{J}_c(0)\ [\mathrm{A/m^2}] = \frac{\hat{J}_S\ [\mathrm{A/m}]}{\delta_c\ [\mathrm{m}]}$$

$$Z_{SR} = j\omega\mu\,\delta_{cR} = \omega\mu\left(\Im\{\delta_{cR}\} + j\Re\{\delta_{cR}\}\right)$$

$$\text{and} \quad v_{pR} = \frac{\omega}{\beta_R} = \omega\frac{|\delta_{cR}|^2}{\Im\{\delta_{cR}\}}\ [\mathrm{m/s}]$$

$$L_{SR} = \mu\Re\{\delta_{cR}\} \quad \text{and} \quad Q_{cR} = \frac{\Re\{\delta_{cR}\}}{\Im\{\delta_{cR}\}} = \left(\frac{\Re\{\delta_{cR}\}}{\Re\{\delta_{co}\}}\right)^2$$

$$Q_{uR}(\omega_{oR})|_{TE_{101}} \cong \frac{Internal\ Volume\ [\mathrm{m^3}]}{Internal\ Surface\ Area\ [\mathrm{m^2}]\cdot\Im\{\delta_{cR}(\omega_{oR})\}[\mathrm{m}]}$$

Classical relaxation-effect model:

$$\delta_{cR} = \Re\{\delta_{co}\}\left(\sqrt{Q_{cR}} - j\frac{1}{\sqrt{Q_{cR}}}\right) = \Im\{\delta_{cR}\}(Q_{cR} - j)$$

$c.f.$

$$Z_{SR} = \Re\{Z_{So}\}\left(\frac{1}{\sqrt{Q_{cR}}} + j\sqrt{Q_{cR}}\right) = \Re\{Z_{SR}\}(1 + jQ_{cR})$$

#5 Resistance Coefficient

Boundary resistance coefficient:

$$k = \frac{\eta_o}{R_s}$$

Surface dissipative power density per unit area:

$$P_S = \frac{P_o}{k}$$ Po = Power flux density per unit area in air

Forward E-field and average power tilt angle:

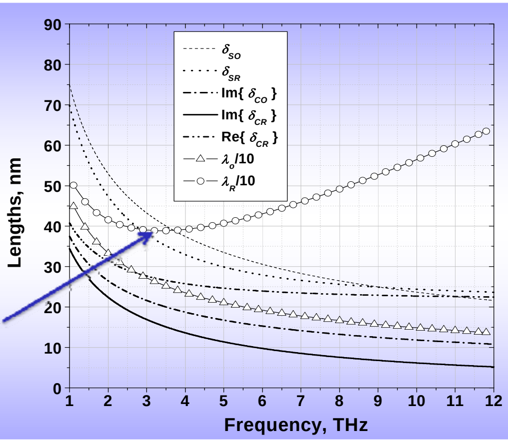
$$\phi = \tan^{-1}(k)$$

For a cube in any mode (with perturbation theory only):

$$Q_U(\omega)\big|_{TE_{mnl}} = \frac{\sqrt{2}\pi}{6}k(\omega) \sim 0.74\, k(\omega) \propto \frac{1}{\sqrt{\omega}}$$

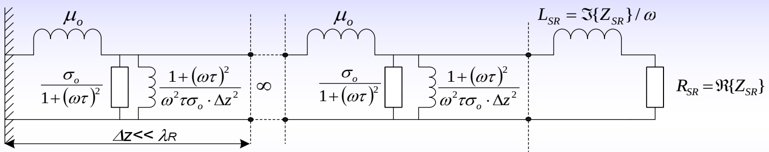
Gold at room temperature

$\omega\tau = 1/\sqrt{3}$



Line Model Validation

$$[ABCD] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Zo\sinh(\gamma l) \\ Yo\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$$



Elementary Lumped-element Circuit

$$[ABCD_1] = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 + j\omega\mu\sigma_R\Delta z^2 & j\omega\mu\Delta z \\ \sigma_R\Delta z & 1 \end{bmatrix}$$

$$\Delta z = \frac{\lambda_R}{N} = \frac{\pi\delta_{So}}{N}Q_{cR}^{3/2}\left[1 + \frac{1}{Q_{cR}^2}\right]$$

When the terminating impedance is equal to the complex conjugate of the characteristic impedance of the transmission line:

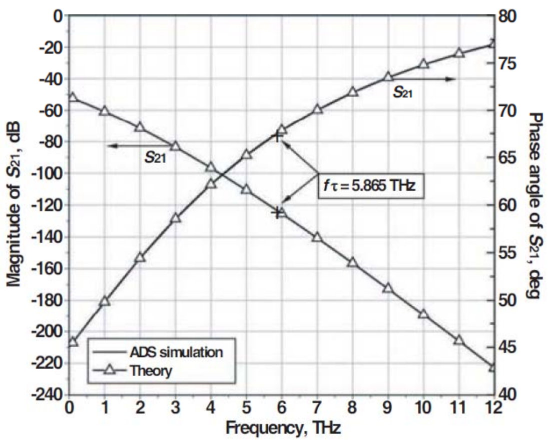
$$S_{11} = S_{22} = \frac{-je^{-2\gamma l}Q_c(1+jQ_c)}{1+(e^{-\gamma l}Q_c)^2} \cong -je^{-2\gamma l}Q_c(1+jQ_c) \quad \text{with} \quad \Re\{\gamma\}l > 2.3$$

$$S_{21} = S_{12} = \frac{e^{-\gamma l}(1+jQ_c)}{1+(e^{-\gamma l}Q_c)^2} \cong e^{-\gamma l}(1+jQ_c) \quad \text{with} \quad \Re\{\gamma\}l > 2.3$$

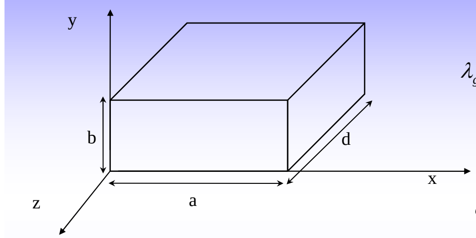
Comparison of modelled parameters for gold at room temperature, at 5.865 THz, determined from theory, direct ABCD parameter matrix calculations and synthesized equivalent transmission line models using commercial circuit simulation software

Parameters for $l = \lambda_R$, $\omega\tau = 1$, $Z_T = Z^*_{SR}$	Theory	Synthesized Transmission Line: ABCD Parameter Matrix Calculations		Synthesized Transmission Line: Microwave Office®	
No. of Sections, N	---	400	800	400	800
$Z_{SR}[\Omega]$	0.46079043 +j1.11244650	0.46079964 +j1.11246875	0.46079043 +j1.11244650	---	---
$\gamma_R\lambda_R[\lambda_R^{-1}]$	15.1689 +j6.2832	15.1681 -j0.0013	15.1685 -j0.0004	---	---
S_{11}	0	0.0006 +j0.0271	0.0002 +j0.0135	0.0006 +j0.0270	0.0001 +j0.0133
Return Loss [dB]	-247.51	-31.35	-37.41	-31.37	-36.9
$S_{11} = e^{-2\pi(1+\sqrt{2})}\cdot[1+j(1+\sqrt{2})]\cdot 10^7$	2.583 +j6.237	2.579 +j6.242	2.582 +j6.240	2.564 +j6.238	2.567 +j6.239
G_{MAX} [dB]	-123.4126	-123.4093	-123.4099	-123.41	-123.4
Absorption Loss [dB]	-131.756	-131.748	-131.752	-131.9	-131.9
$\forall S_{11} = \tan^{-1}(1+\sqrt{2})[°]$	67.50	67.55	67.52	67.58	67.56

Gold at room temperature



THz MPRWGs



$$\lambda_{g\_ideal} = \frac{\lambda_o}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}} = \frac{\lambda_o}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

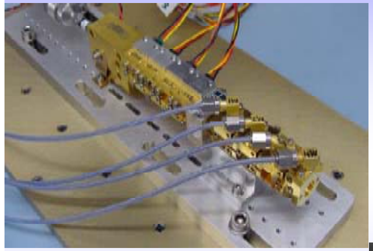
$$\alpha_{WG}\big|_{TE_{10}} = \frac{Rs(\omega)}{\eta_o} G_{10}(\omega) \quad [Np/m]$$

$$Zs(\omega) = \sqrt{\frac{j\omega\mu_o\mu_r}{\sigma + j\omega\varepsilon_o}} \quad [\Omega/square]$$

and

$$G_{10}(\omega) = \frac{1+2\frac{b}{a}\left(\frac{\omega_c|_{TE_{10}}}{\omega}\right)^2}{b\sqrt{1-\left(\frac{\omega_c|_{TE_{10}}}{\omega}\right)^2}} \quad [m^{-1}]$$

New Waveguide Standard for Terahertz Frequencies

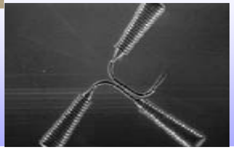


1.5 THz Frequency Multiplier Chain (JPL)

John Ward, JPL 2006

Why New Standard ?

- No extension of industry standards above 325 GHz (WR-3)
- No compatibility among hardware
- Impractical transitions
- Defined in inches, rounding errors



5 THz horns and coupler (C. Walker U. Arizona)

Proposed air-filled MPRWG and cavity resonator definitions and specifications

ISO 497 Preferred Metric Size: B	ISO 497 Preferred Metric Size: i	Proposed Frequency Band Designation (µm): JPL	Proposed Frequency Band Designation (µm): Our	Internal Dimensions $a \times b$ (µm²)	TE_{10}-mode Ideal Cutoff Frequency f_C (THz)	Lower-Band Frequency Factor f_L / f_C	Mid-Band Frequency $f_O = 1.55 f_C$ (THz)	Upper-Band Frequency Factor f_U / f_C	Useful Frequency Range $f_L \to f_U$ (THz)	TE_{101}-mode Ideal Cavity Resonance Frequency $f_{101\_ideal} = \sqrt{1.5} f_c$ (THz)
20'	6	200		200×100	0.75	1.20	1.162	1.93	0.90→1.45	0.919
20'	4	160		160×80	0.94	1.23	1.452	1.92	1.15→1.80	1.148
20'	2	125		125×62.5	1.20	1.21	1.860	1.92	1.45→2.30	1.470
20'	0	100		100×50	1.50	1.20	2.325	1.93	1.80→2.90	1.837
40	35		75	75×32.5	2.00	1.20	3.100	1.90	2.40→4.00	2.449
20'	14		50	50×25	3.00	1.20	4.650	1.90	3.60→5.00	3.674
20'	10		32	32×16	4.68	1.20	7.260	1.90	5.62→8.90	5.732
20'	8		25	25×12.5	6.00	1.20	9.300	1.90	7.20→12.00	7.348

New Waveguide Standard for Terahertz Frequencies: ISO 497 Preferred Metric Sizes

- Widely used global industry standard
- Logarithmic scale
- Infinitely extendable
- Repeats every decade
- Decision tree with range of coarse and fine spacing

ISO 497 Preferred Metric Sizes

1st Choice	2nd Choice	3rd Choice
1		1.1
	1.25	1.4
1.6		1.8
	2	2.2
2.5		2.8
	3.2	3.6
4		4.5
	5	5.6
6.3		7.1
	8	9
10		

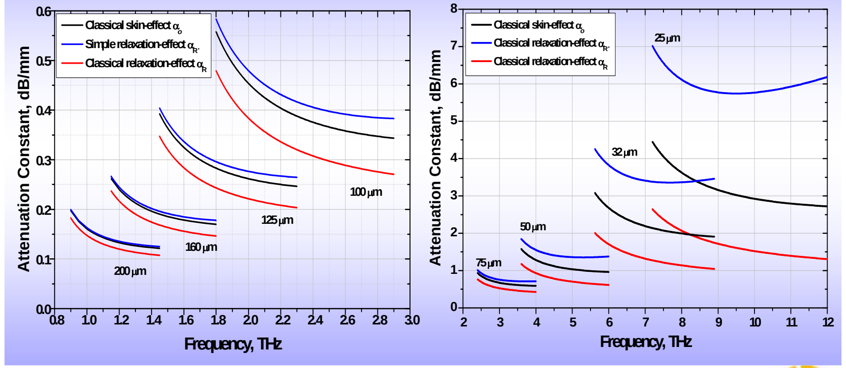
Variational Methods for Calculating Propagation Constant for the TE_{mo} Mode

$$\gamma_{mo}^2 = \Gamma_d^2 - j\frac{2Z_s}{\omega\mu_o\mu_r b}\left[\left(\frac{\Gamma_d}{k_c}\frac{m\pi}{a}\right)^2 - k_c^2\left(1+\frac{2b}{a}\right)\right]; \quad \Gamma_d^2 = k_c^2 - k_{od}^2$$

$$k_c = \omega_c\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r} \rightarrow \omega_c\sqrt{\mu_o\varepsilon_o} = \frac{\omega_c}{c} \quad \text{in free space}$$

$$k_{od} = \omega\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r\left(1 - j\tan\delta\right)} \rightarrow k_o = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} \quad \text{in free space}$$

Calculated attenuation constants for the dominant TE_{10} mode



Errors in attenuation constant

Classical skin-effect: 108% error at 12 THz

$$E_{\alpha_o}(\omega) = \left(\frac{\alpha_{WGo} - \alpha_{WGR}}{\alpha_{WGR}}\right)\cdot 100\,\% \cong \left[\sqrt{\sqrt{1+(\omega\tau)^2} + \omega\tau} - 1\right]\cdot 100\,\%$$

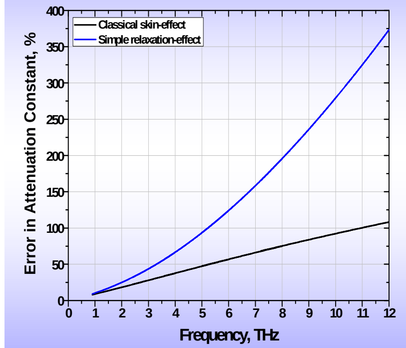
$$E_{\alpha_o}(\omega) \cong \left(\frac{R_{So} - R_{SR}}{R_{SR}}\right)\cdot 100\,\% \quad \text{with} \quad Z_{SR} = R_{So}\left[\frac{1}{(1+\xi\,\omega\tau)} + j(1+\xi\,\omega\tau)\right]$$

$$\therefore E_{\alpha_o}(\omega) = \left(\sqrt{Q_{cR}} - 1\right)\cdot 100\% = \xi\, Q_{mR}\cdot 100\%$$

$$\therefore E_{\alpha_o}(\omega) \approx 0.539\, Q_{mR}\cdot 100\% \quad \text{for} \quad 0 \le \omega\tau \le 2$$

e.g. Calculated error using this approximation is 110% at 12 THz

Resulting errors in attenuation constant calculations



Power Loss Method (for simplicity)

$$\alpha = \alpha_c + \alpha_d$$

$$\alpha_c = \frac{R_s}{b\,\eta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[1 + 2\left(\frac{b}{a}\right)\left(\frac{f_c}{f}\right)^2\right]$$

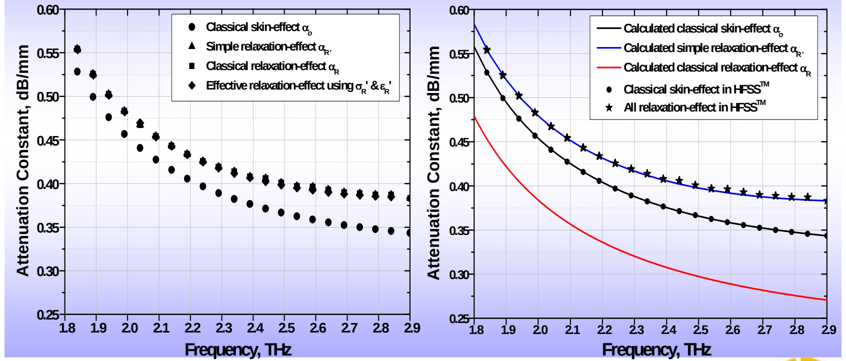
$$E_{\alpha_{R'}} = \left(\frac{\alpha_{R'} - \alpha_R}{\alpha_R}\right)\cdot 100\% \cong \left[\sqrt{\frac{1+(\omega\tau)^2}{\sqrt{1+(\omega\tau)^2} - \omega\tau}} - 1\right]\cdot 100\%$$

Classical skin-effect: 108% error at 12 THz

$$E_{\alpha_o} = \left(\frac{\alpha_o - \alpha_R}{\alpha_R}\right)\cdot 100\% \cong \left[\sqrt{\sqrt{1+(\omega\tau)^2} + \omega\tau} - 1\right]\cdot 100\%$$

Simple relaxation-effect: 373% at 12 THz

HFSS™ electromagnetic simulations of attenuation constant for JPL 100 µm band



THz Cavity Resonators

With Perturbation model, unloaded Q-factor in the mnl mode :

$$Q_U(\omega'_o)\big|_{TE_{mnl}} = \frac{\omega_{I\_mnl}}{R_S(\omega'_o)}\Gamma_{mnl} \quad \text{where } \Gamma_{mnl} = \mu_o \psi_{mnl} \quad [H]$$

Geometric factor : $\psi_{mnl} = \frac{\lambda_{I\_mnl}}{8}\left[\frac{2b(a^2+d^2)^{3/2}}{2b(a^3+d^3)+ad(a^2+d^2)}\right] \quad [m]$

$$\therefore Q_U(\omega'_o)\big|_{TE_{mnl}} = k(\omega'_o)\frac{\pi}{2}\left[\frac{b(a^2+d^2)^{3/2}}{2b(a^3+d^3)+ad(a^2+d^2)}\right] \propto (\omega'_o)^{-1/2}, \text{ where } k(\omega'_o) = \frac{\eta_o}{R_S(\omega'_o)}$$

$$\text{For a cube, } Q_U(\omega'_o)\big|_{TE_{mnl}} = \frac{\sqrt{2}\pi}{6}k(\omega'_o) \sim 0.74\, k(\omega'_o)$$ For all modes!

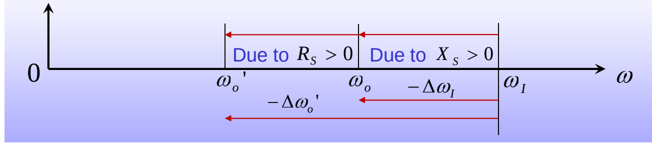
Complex Resonant Frequency

As determined by Eigenmode solution in HFSS™

$$\tilde{\omega}_o \equiv \omega_o' + j\omega_o'' = \omega_o\sqrt{1-\left(\frac{1}{2Q_u(\omega_o)}\right)^2} + j\frac{\omega_o}{2Q_u(\omega_o)}$$

$$\tilde{\omega}_o \sim \omega_o + j\frac{\omega_o}{2Q_u(\omega_o)} \text{ with perturbation model}$$

Overall frequency detuning $-\Delta\omega_o' = (\omega_I - \omega_o')$
takes into account both perturbation $-\Delta\omega_I = (\omega_I - \omega_o)$
(represented by Xs) and detuning due to Ohmic losses (represented by Rs)



$$\lambda_{I\_mnl} = \frac{c}{f_{I\_mnl}} \qquad f_{I\_mnl} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2} \equiv \frac{\omega_{I\_mnl}}{2\pi}$$

$$\lambda_{I\_101} = \frac{2ad}{\sqrt{a^2+d^2}} = \begin{cases} \sqrt{2}\,a & for \quad half-height \quad i.e. \quad d=a=2b \\ \sqrt{8/3}\,a & most \quad common \quad d=\sqrt{2}\,a = 2\sqrt{2}\,b \end{cases}$$

$$\psi_{101} = \frac{abd(a^2+d^2)}{2[2b(a^3+d^3)+ad(a^2+d^2)]} = \begin{cases} Volume/Ara \quad for \quad d=a \\ a/6 \quad for \quad cube \quad i.e. \quad d=a=b \\ a/8 \quad for \quad half-height \quad i.e. \quad d=a=2b \\ 3a/[2(\sqrt{2}+10)] \quad most \quad common \quad i.e. \quad d=\sqrt{2}\,a=2\sqrt{2}\,b \end{cases}$$

Over-simplified Solution

(O. Klein et al.,1993, and M. Dressel & G. Gruner, 2002)

$$\omega_I \longrightarrow R_S(\omega_I) \longrightarrow Q_U(\omega_I) = \frac{\omega_I\Gamma}{R_S(\omega_I)} \longrightarrow \tilde{\omega}_o \sim \omega_I + j\frac{\omega_I}{2Q_u(\omega_I)}$$

Simplified Solution

(J. C. Salter, 1946)

$\tilde{\omega}_o \sim (\omega_I + \Delta\omega_I) + j\frac{\omega_I}{2Q_u(\omega_I)}$ where $\Delta\omega_I$ is due to perturbation

Cavity Perturbation Formula

Change in Surface Impedance:

$$\Delta Z_S = -j2\Gamma \cdot \Delta\tilde{\omega}_o$$

Exact Solution

$$\left.\begin{matrix}\omega_I \\ \boxed{\omega_o}\end{matrix}\right\} \longrightarrow \boxed{Q_U(\omega_o)} \longrightarrow \tilde{\omega}_o \equiv \omega_o' + j\omega_o'' = \omega_o\sqrt{1-\left(\frac{1}{2Q_u(\omega_o)}\right)^2} + j\frac{\omega_o}{2Q_u(\omega_o)}$$

Self-consistent analytical solution for all dispersion models

Self-consistent analytical solution With $\mu_r \neq 1$ and $\omega\tau > 0$

Using $\mathrm{Im}\{\delta_c(\omega_o)\}$ instead of δ_S

$$\omega_o' \longrightarrow R_S(\omega_o') \longrightarrow Q_U(\omega_o')$$

$$E_\omega \qquad\qquad E_Q$$

Self-consistent Solution for General Case of Metal

$$Q_u(\omega_o) = \frac{\lambda_I(\omega_I)}{8\mu_r\Im\{\delta_c(\omega_o)\}}\left(\frac{\omega_I}{\omega_o}\right)\left\{\frac{2b\left(a^2+d^2\right)^{\frac{3}{2}}}{\left[2b\left(a^3+d^3\right)+ad\left(a^2+d^2\right)\right]}\right\}$$

$$\therefore Q_u(\omega_o) = \frac{1}{\mu_r\Im\{\delta_c(\omega_o)\}}\left(\frac{\omega_I}{\omega_o}\right)\left(\frac{3\sqrt{2}a}{4\left(5\sqrt{2}+1\right)}\right) \quad \text{for most common i.e. } d=\sqrt{2}a=2\sqrt{2}b$$

Derivation of Driven Frequency of Oscillation

After equalizing equations for surface reactance:

$$\omega_o \rightarrow \omega_{oR}$$

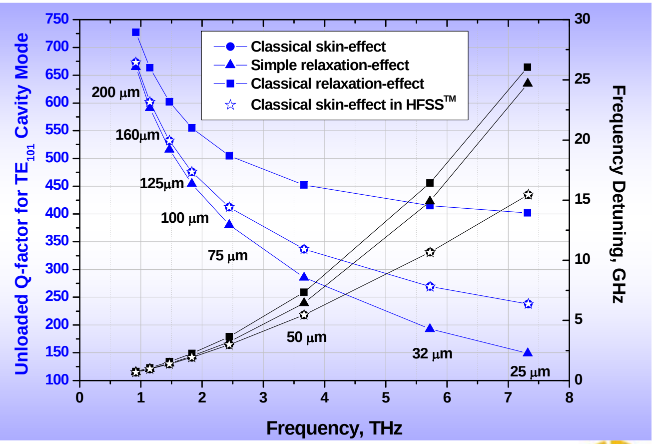
$$\frac{\sqrt{\omega_{oR}\left(\sqrt{1+(\omega_{oR}\tau)^2}+\omega_{oR}\tau\right)}}{(\omega_I-\omega_{oR})} - K \equiv 0$$

$$\text{where } K = \sqrt{\frac{8\mu_o\sigma_o}{\mu_r}} \cdot \left(\frac{3a}{2(\sqrt{2}+10)}\right) \quad \text{for most common i.e. } d=\sqrt{2}a=2\sqrt{2}b$$

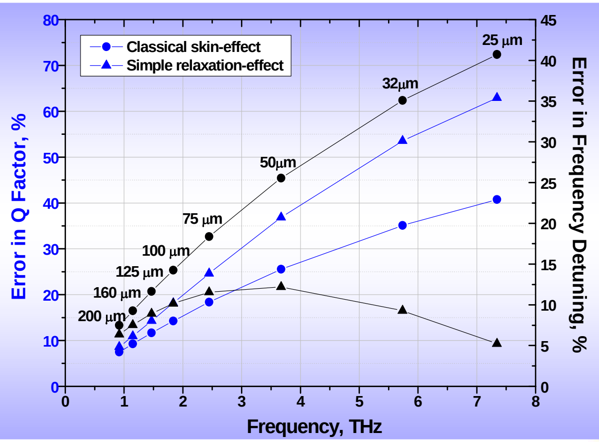
$$\omega_o \rightarrow \omega_{oR'} \qquad \frac{\sqrt{\omega_{oR'}\left(1+(\omega_{oR'}\tau)^2\right)}}{(\omega_I-\omega_{oR'})} - K \equiv 0$$

$$\omega_o \rightarrow \omega_{oo} \qquad \frac{\sqrt{\omega_{oo}}}{(\omega_I-\omega_{oo})} - K \equiv 0 \qquad \therefore \omega_{oo} = (W+\omega_I) - \sqrt{(W+\omega_I)^2-\omega_I^2} \quad \textbf{where } W=\frac{1}{2K^2}$$

Gold at room temperature



Gold at room temperature



Errors in Unloaded Q-factor

Classical skin-effect: 41% error at 7.3 THz

$$E_{Qo}(\omega'_{oR}) = \left|\frac{Q_{Uo}(\omega'_{oo}) - Q_{UR}(\omega'_{oR})}{Q_{UR}(\omega'_{oR})}\right| \times 100\,\% = \left|\sqrt{\frac{Q_{mR}(\omega'_{oR})}{Q_{mR}(\omega'_{oo})Q_{cR}(\omega'_{oR})}} - 1\right| \times 100\,\%$$

$$E_{Qo}(\omega'_{oR}) \cong \left|\frac{1}{\sqrt{Q_{cR}(\omega'_{oR})}} - 1\right| \times 100\ \% \qquad \omega'_{oR} \cong \omega'_{oo}$$

$$\therefore E_{Qo}(\omega'_{oR}) \approx \left(\frac{1}{1 + 1.8553\, Q_{mR}(\omega'_{oR})^{-1}}\right) \times 100\,\% \qquad 0 \le \omega\tau \le 2$$

e.g. Calculated error using this approximation is 40% at 7.3 THz

$$Q_{Uo}(\omega'_{oo})\Big|_{TE_{mnl}} = \frac{2}{\mu_r \delta_{So}(\omega'_{oo})}\left(\frac{\omega_{I\_mnl}}{\omega'_{oo}}\right)\psi_{mnl} \longrightarrow Q_{Uo}(\omega'_{oo})\Big|_{TE_{mnl}} = \frac{Q_{Uo}(\omega\tau = 1)\Big|_{TE_{mnl}}}{\sqrt{Q_{mR}(\omega'_{oo})}}$$

$$Q_{UR}(\omega'_{oR})\Big|_{TE_{mnl}} = \frac{1}{\mu_r \Im\{\delta_{cR}(\omega'_{oR})\}}\left(\frac{\omega_{I\_mnl}}{\omega'_{oR}}\right)\psi_{mnl} \longrightarrow Q_{UR}(\omega'_{oR})\Big|_{TE_{mnl}} = Q_{Uo}(\omega\tau = 1)\Big|_{TE_{mnl}} \sqrt{\frac{Q_{cR}(\omega'_{oR})}{Q_{mR}(\omega'_{oR})}}$$

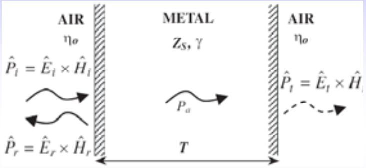
- HFSS[TM] current versions (v.10 & 11) cannot accurately predict the performance of structures operating at terahertz frequencies
- Intrinsic dispersion models: Classical skin-effect & simple relaxation-effect inflate the attenuation. Therefore, extrinsic loss effects (e.g. surface roughness) may be underestimated

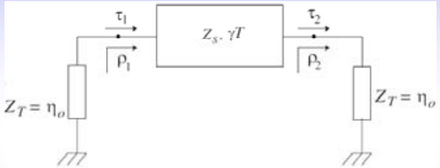
THz Metal Shielding

RF metal shielding is found in many applications; ranging from:

- Construction of high isolation subsystem partitioning walls
- Efficient quasi-optical components (e.g. planar mirrors and parabolic reflectors for open resonators and antennas)
- Creating guided-wave structures that have (near-)zero field leakage (e.g. metal-pipe rectangular waveguides and associated closed cavity resonators)
- Embedding ground planes within compact 3D multi-layer architectures.

Uniform plane wave at normal incidence to an infinite single planar shield in air





Physical representation | Equivalent 2-port network model

Voltage-wave reflection coefficients: $\rho_1 = \dfrac{Z_S - Z_T}{Z_S + Z_T}$ $\rho_2 = \dfrac{Z_T - Z_S}{Z_S + Z_T} \equiv -\rho_1$

Voltage transmission coefficients: $\tau_1 \equiv 1 + \rho_1 = \dfrac{2Z_S}{Z_S + Z_T}$ $\tau_2 \equiv 1 + \rho_2 = \dfrac{2Z_T}{Z_S + Z_T}$

✍ Metal shields should be made as thin as possible, while meeting the minimum values for figures of merit within the intended bandwidth of operation, in order to reduce weight and cost.

✍ For reasons of structural integrity, thin metal shielding can be deposited onto either a solid plastic/ceramic or even honeycomb supporting wall.

✍ Thin metal shielding embedded between dielectric layers (e.g. to create conformal ground planes or partition walls) can avoid issues of poor topography when integrating signal lines within 3D multi-layered architectures.

✍ Shielding effectiveness, return loss and absorptance (or absorptivity) are important figures of merit that are quoted to quantify the ability to shield electromagnetic radiation.

✍ An electrical engineering approach, which can include network analysis and the synthesis of predictive equivalent transmission line models, can accurately solve specific EM problems.

S-Parameters Analysis for Single Planar Shield

Transient response solution

$$S_{21} = \tau_1 \cdot \left[e^{-\gamma T} \cdot \sum_{i=0}^{\infty} \left(e^{-\gamma T} \rho_2 \right)^{2i} \right] \cdot \tau_2 \qquad S_{11} = \rho_1 + \tau_1 \cdot \left[e^{-2\gamma T} \rho_2 \cdot \sum_{i=0}^{\infty} \left(e^{-\gamma T} \rho_2 \right)^{2i} \right] \cdot \tau_2$$

$$i \in [0, 1, 2..\infty]$$

Steady-state solution

$$S_{21} = \tau_1 \cdot \left[\frac{e^{-\gamma T}}{1 - \left(e^{-\gamma T} \rho_1 \right)^2} \right] \cdot \tau_2 \qquad S_{11} = \rho_1 \cdot \left[\frac{1 - e^{-2\gamma T}}{1 - \left(e^{-\gamma T} \rho_1 \right)^2} \right]$$

$$\left| e^{-\gamma T} \rho_1 \right| < 1$$

S-Parameters Analysis for Single Planar Shield

Boundary resistance coefficient $k = \frac{\eta_o}{R_S} >> Q_c \geq 1$

$$\rho_1 = \frac{(1-k) + jQ_c}{(1+k) + jQ_c} \approx 1 \qquad \tau_1 = \frac{2(1+jQ_c)}{(1+k) + jQ_c} \cong \frac{2(1+jQ_c)}{k + jQ_c} \approx \frac{2(1+jQ_c)}{k} \qquad \tau_2 = \frac{2k}{(1+k) + jQ_c} \cong \frac{2k}{k + jQ_c} \approx 2$$

a = thickness normalized to normal skin depth

$-\gamma T \rightarrow -\gamma \cdot a\,\delta_S = -a \cdot \left(1 + \frac{j}{Q_c}\right)$ and $T = a_R\,\delta_{SR}$

Therefore, $\gamma_R \cdot 5\delta_{SR} = 5\frac{\gamma_R}{\Im\{\gamma_R\}Q_{cR}} = 5\left(1 + \frac{j}{Q_{cR}}\right) \quad [(5\delta_{SR})^{-1}]$

e.g. $Q_{cR}(\omega\tau = 1) = (1 + \sqrt{2}) \longrightarrow \gamma_R(\omega\tau = 1) \cdot 5\delta_{SR}(\omega\tau = 1) = 5\left(1 + \frac{j}{1+\sqrt{2}}\right) \quad [(5\delta_{SR})^{-1}]$

S-Parameters Analysis: Classical Skin-effect Model

$$\boxed{a_o = \frac{T}{\delta_{So}} = a_R\left(\frac{\delta_{SR}}{\delta_{So}}\right)}$$

$$S_{21o} = \frac{4\sqrt{2j}\,k_o \cdot e^{-a_o\sqrt{2j}}}{\left(\sqrt{2j} + k_o\right)^2 - \left(\sqrt{2j} - k_o\right)^2 \cdot e^{-2a_o\sqrt{2j}}} \approx \frac{2\sqrt{2j}}{k_o \cdot \sinh\left(a_o\sqrt{2j}\right)}$$

Approximation error : < 0.6% up to a_R = 10, frequency less than 12 THz

$$S_{11o} = \rho_{1o} \cdot \left[\frac{1 - e^{-2a_o\sqrt{2j}}}{1 - \left(e^{-a_o\sqrt{2j}}\rho_{1o}\right)^2}\right] = \frac{\sinh\left(a_o\sqrt{2j}\right)}{\sinh\left[a_o\sqrt{2j} - \ln\left(\frac{\sqrt{2j} - k_o}{\sqrt{2j} + k_o}\right)\right]}$$

S-Parameters Analysis: Classical Relaxation-effect Model

$$S_{21R} = \frac{4k_R \cdot (1 + jQ_{cR}) \cdot e^{-a_R\left(1+\frac{j}{Q_{cR}}\right)}}{[(1+k_R) + jQ_{cR}]^2 - [(1-k_R) + jQ_{cR}]^2 \cdot e^{-2a_R\left(1+\frac{j}{Q_{cR}}\right)}} \approx \frac{2(1 + jQ_{cR})}{k_R \cdot \sinh\left[a_R\left(1 + \frac{j}{Q_{cR}}\right)\right]}$$

Approximation error : < 0.4% up to a_R = 10, frequency less than 12 THz

$$S_{11R} = \rho_{1R} \cdot \left[\frac{1 - e^{-2a_R\left(1+\frac{j}{Q_{cR}}\right)}}{1 - \left(e^{-a_R\left(1+\frac{j}{Q_{cR}}\right)}\rho_{1R}\right)^2}\right] = \frac{\sinh\left[a_R\left(1 + \frac{j}{Q_{cR}}\right)\right]}{\sinh\left[a_R\left(1 + \frac{j}{Q_{cR}}\right) - \ln\left(\frac{(1-k_R) + jQ_{cR}}{(1+k_R) + jQ_{cR}}\right)\right]}$$

Analysis of Shielding Effectiveness (SE)

$$SE_{dB} = -10\log_{10}\left(\frac{P_t}{P_i}\right) \quad [dB]$$

$$SE_{dB} = -20\log_{10}\left|S_{21R} \cdot e^{+j\beta_o T}\right| = -20\log_{10}\left|S_{21R}\right| \equiv R_{dB} + A_{dB} + M_{dB}$$

Cross-boundary reflections

$$R_{dB} = -20\log_{10}\left|\tau_{1R} \cdot \tau_{2R}\right| = 20\log_{10}\left|\frac{(Z_{SR} + \eta_o)^2}{4Z_{SR}\eta_o}\right| \sim 20\log_{10}\left|\frac{\eta_o}{4Z_{SR}}\right| \rightarrow 20\log_{10}\left|\frac{k_R}{4(1 + jQ_{cR})}\right|$$

Absorption

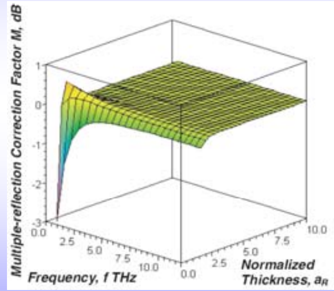
$$A_{dB} = -20\log_{10}\left|e^{-\gamma_R T}\right| = 20\log_{10}\left(e^{T/\delta_{SR}}\right) \rightarrow 20\log_{10}\left(e^{a_R}\right) \cong 8.686\,a_R$$

Multiple reflections

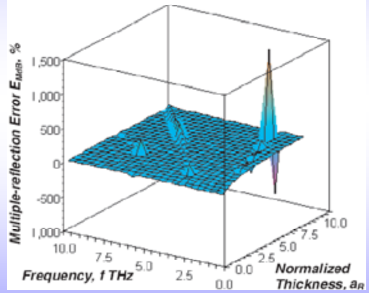
$$M_{dB} = 20\log_{10}\left|1 - \left(e^{-\gamma_R T}\rho_{1R}\right)^2\right| \sim 20\log_{10}\left|1 - e^{-2\gamma_R T}\right| \rightarrow 20\log_{10}\left|1 - e^{-2a_R\cdot\left(1+\frac{j}{Q_{cR}}\right)}\right|$$

Correction Factor for Multiple Reflections Calculations

$$M_{dB} = 20\log_{10}\left|1-\left(e^{-\gamma_R T}\rho_{1R}\right)^2\right| \qquad E_{MdB} = \frac{M_{dB\_approximation} - M_{dB\_exact}}{M_{dB\_exact}}\cdot 100\%$$

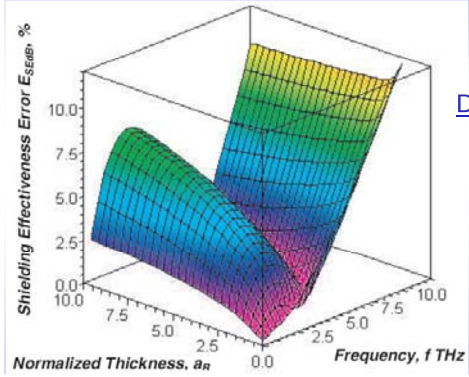


Classical-relaxation Model



Error using Approximation

Screening Effectiveness Calculations-Error using Approximation



$$E_{SE\,dB} = \left|\frac{SE_{dBo} - SE_{dBR}}{SE_{dBR}}\right|\cdot 100\%$$

Due to absorption, a peak value

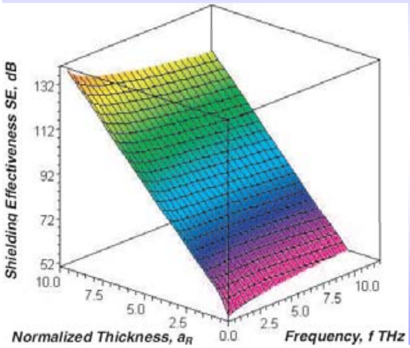
7.8% at a_R = 10 $\omega\tau = 1/\sqrt{3}$

Beyond the peak region

Linearly increase,
11.7% at T = l_m = 1.6δ_{SR}

$$\omega\tau \cong 2.046$$

Screening Effectiveness Calculations



Classical Relaxation-effect Model

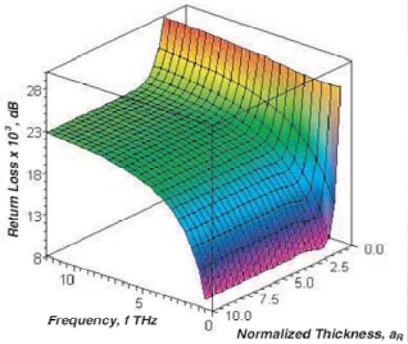
$$SE_{dBR} \cong 10\log_{10}\left(\frac{8\left(1+Q_{cR}^2\right)/k_R^2}{\cosh(2a_R)-\cos(2a_R/Q_{cR})}\right)$$

Classical Skin-effect Model

$$SE_{dBo} \cong 10\log_{10}\left(\frac{16/k_o^2}{\cosh(2a_o)-\cos(2a_o)}\right)$$

Less shielding as
frequency increases and/or
thickness decrease

Reflection Characteristics: Return Loss Calculations

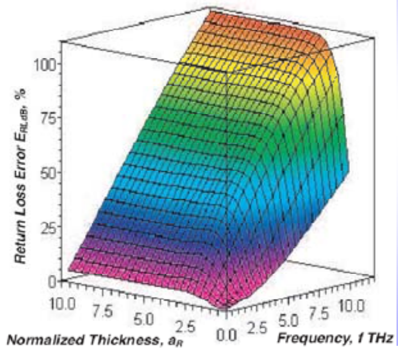


$$RL_{dB} = -10\log_{10}\left(\frac{P_r}{P_i}\right) = -10\log_{10}(\Gamma) = -20\log_{10}|S_{11}| \quad [dB]$$

$$RL = \Gamma_R = \begin{cases} |S_{11R}|^2 & \textit{for all } a_R \\ \cong |\rho_{1R}|^2 \approx \dfrac{k_R-2}{k_R+2} & \textit{for } a_R > 3 \end{cases}$$

Less reflected power as
frequency increases and/or
thickness decrease

Reflection Characteristics: Return Loss Calculations Error using Classical Skin-effect



$$E_{RL\,dB} = \left|\frac{RL_{dBo} - RL_{dBR}}{RL_{dBR}}\right| \cdot 100\%$$

109% at T = 10δ_{SR}

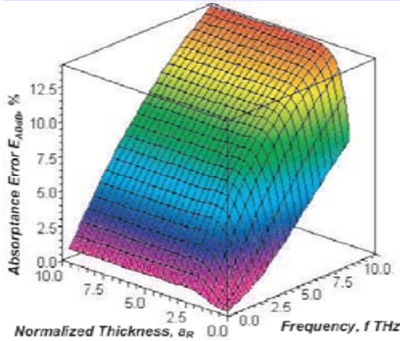
$$\omega\tau \cong 2.046$$

Thickness invariant above

$$a_R \approx 3$$

Radio Frequency Engineering
Lecture #6 Engineering Approach

Absorptance Calculations: Error using Classical Skin-effect



$$E_{ABdB} = \left|\frac{AB_{dBo} - AB_{dBR}}{AB_{dBR}}\right| \cdot 100\%$$

14% at T = 10δ_{SR}

$$\omega\tau \cong 2.046$$

Thickness invariant above

$$a_R \approx 3$$

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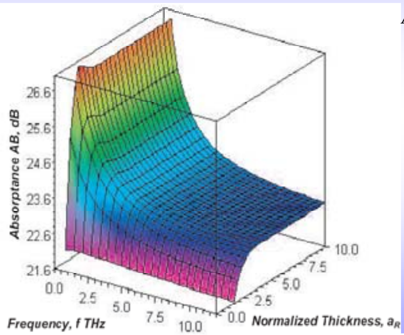
Absorptance Calculations



$$AB_{dB} = -10\log_{10}\left(\frac{P_a}{P_i}\right) = -10\log_{10}\left[1 - \left(\frac{P_t}{P_i}\right) - \left(\frac{P_r}{P_i}\right)\right]$$

$$= -10\log_{10}(1 - SE - RL)$$

$$AB = \begin{cases} 1 - |S_{21R}|^2 - |S_{11R}|^2 & \text{for all } a_R \\ \cong 1 - |\rho_{1R}|^2 \approx \dfrac{4}{k_R} \propto R_{SR} & \text{for } a_R > 3 \end{cases}$$

More absorbed power as frequency increases and/or thickness decrease

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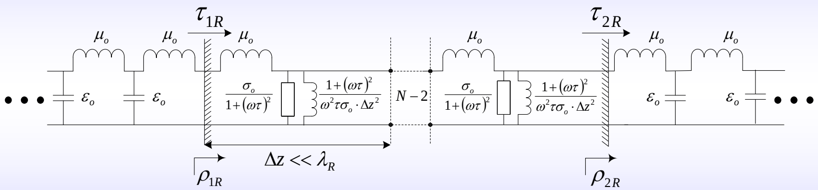
Radio Frequency Engineering
Lecture #6 Engineering Approach

Synthesis Modelling of Metal Shielding Walls

Classical Relaxation-effect Model: Equivalent transmission line model



For example, at 5.865 THz, gold at room temperature

$$L_R \cdot \Delta z = 1.341\,[fH] \qquad G_R \cdot \Delta z = 24.1\,[mS] \qquad L_{SHUNT\_R} \cdot \Delta z = 1.126\,[pH]$$

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Comparison of modelled parameters for gold at room temperature, at 5.865 THz, determined from theory, direct ABCD parameter matrix calculations and synthesized equivalent transmission line models using commercial circuit simulation software

Parameters $a_R = 5,\ \omega\tau = 1,\ Z_T = \eta_o$	**Theory**	**Synthesized Transmission Line** (*N* = 400 *sections per wavelength*) — *ABCD* **Parameter Matrix Calculations**		**Microwave Office®**	
		Value	Error [%]	Value	Error [%]
Z_{SR} [Ω]	0.4607904263 +j1.112446496	0.4607904255 +j1.112446495	-1.7×10^7 -0.9×10^7	---	---
$\gamma_R \cdot 5\delta_{SR}$ [$5\delta_{SR}^{-1}$]	5.0000 +j2.0711	5.0242 +j2.0807	+0.484 +0.464	---	---
$S_{21R} \cdot 10^5$	5.349 −j6.725	5.309 −j6.699	−0.748 −0.387	5.308 −j6.697	−0.766 −0.416
∇S_{21R} [°]	−51.499	−51.602	+0.200	−51.6	+0.196
Screening Effectiveness [dB]	81.317	81.363	+0.057	81.36	+0.053
S_{11R}	−0.9975 +j0.0059	−0.9975 +j0.0060	0.000 +1.695	−0.9975 +j0.0060	0.000 +1.695
Return Loss [dB]	0.02124	0.02124	0.000	0.02124	0.000

Conclusions

- **An electrical engineering approach to the modelling of specific electromagnetic problems at THz frequencies is introduced, using 5 interrelated concepts (transmission line modelling, kinetic inductance, Q-factor, complex skin depth and boundary resistance coefficient).**

- **The *Engineering Approach* has 5 main advantages:**
  - **Excellent pedagogical tool**
  - **Greatly reduces otherwise lengthy mathematical derivations**
  - **Reduces risk of introducing mistakes**
  - **Avoids the need for poor approximations**
  - **Can replace the need for slow numerical computations (with simple structures)**
  - **Gives new perspectives and deeper insight**

- **While the focus has been on the characterization of normal metals (magnetic and non-magnetic) at room temperature, it is believed that the same methodology may also be applied to metals operating in anomalous frequency-temperature regions, superconductors, semiconductors, carbon nanotubes and metamaterials.**